Chapter 9

Tidal Gravity

that tends to
stretch and
squeeze

Tidal gravity is most commonly known as the cause of the tides of the oceans on Earth. The Sun and the Moon both tugs on the Earth and cause a bulge on the surface of the Earth in some places and a flattening at other places. How the tides work will be discussed later, but first, what is tidal gravity?

Loosely speaking, tidal gravity manifests itself as unequal gravitational accelerations on different parts of a free-falling body, caused by the different distances and angles of the body parts from the centre of the source of gravity. We will first look at tidal gravity from the perspective of Newton's theory and then discuss the general relativistic interpretation of tidal gravity.

9.1 Newtonian tidal gravity

Consider what happens if an object with a diameter d free-falls straight towards a large spherical mass M. Let the centre of the falling object be a radial distance r from the centre of M.

The object's side nearest to the mass experiences a gravitational attraction that is larger than the gravitational attraction experienced by the centre of the object. Likewise the centre of the object experiences a gravitational attraction that is larger than the gravitational attraction experienced by the far side of the object, causing a stretching effect in the radial direction.

The stretching acceleration of the near side of the falling object relative to it's centre is (in geometric units)

$$\frac{-GM}{(r-d/2)^2} - \frac{-GM}{r^2} \approx \frac{-2GMd}{r^3}, \text{ if } r \gg d.$$

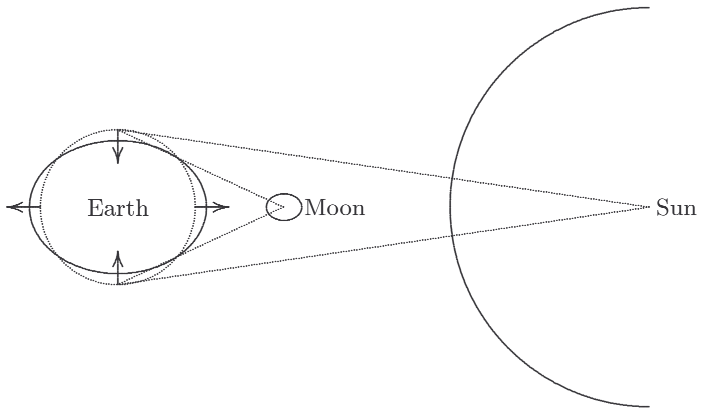

Figure 9.1: Tidal forces from the Sun and the Moon tend to stretch and squeeze the free falling Earth. The Moon's crust suffers the same effects. The magnitudes of the stretching and squeezing are relatively small and contrary to popular believe, they are not the direct cause of the tides in the oceans of the Earth. See text.

Likewise, the stretching acceleration of the far side relative to the centre is

$$\frac{GM}{(r+d/2)^2} - \frac{-GM}{r^2} \approx \frac{2GMd}{r^3}, \text{ if } r \gg d.$$

Because the gravitational acceleration works towards the centre of the massive object, there is also an active flattening effect in the transverse direction, as viewed from mass M (figure 9.1). The magnitude of the 'squeeze' effect is very closely the same as the stretching effect.

It is clear that because of the cubed distance denominator, tidal gravity falls off very rapidly with distance. This is significant in the case of the tides in Earth's oceans. Although the Sun has a much larger gravitational attraction on Earth than the Moon has on the Earth, the Moon produces the larger tidal effect on Earth, simply because it is so much closer to us.

The tidal stretching produced on Earth by the Moon and the Sun can be viewed as four acceleration vectors, i.e. a near side and a far side vector for the Moon and likewise, two for the Sun. The four vectors rotate around Earth (relative to it's surface) at different rates and with changing orientations and magnitudes.

To simplify things, we will analyse the tides with the Sun and the Moon roughly lined up (as at the new moon) and use the direction of their combined gravitational force as a reference, which we will refer to as the *tidal vector*.

The fact that the Earth is in orbit around the Sun and not free falling directly towards it, does not change the situation in any significant way. Any orbit is just a free-fall with some transverse velocity added.

This is all very easy as far as the Sun is concerned, but we know that the Moon is in fact the major contributor to tidal gravity on Earth. It is not so intuitively clear that the Earth is also free-falling relative to the Moon.

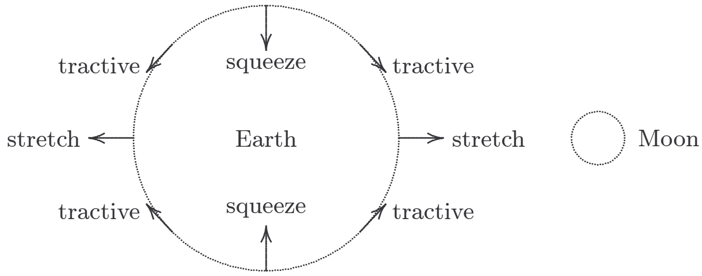

Figure 9.2: At points between the forces of pure stretch and pure squeeze, both stretching and squeezing are at work. This results in "tractive" force components working along the surface of the Earth, capable of dragging water horizontally.

Look at it like this: park the Earth and the Moon at an arbitrary distance from each other in some inertial reference frame and then set them free. Each will start to accelerate towards the other, as measured in the inertial reference frame.

While the Earth is orbiting the Sun, it is continously falling towards the Moon as well. More correctly stated, the Earth and the Moon orbit each other around their common centre of gravity (the barycentre), while the common centre of gravity orbits the Sun. Also see the box on page 138 for an interesting aside on the moon's orbit. Now back to tidal gravity and specifically the effect on the oceans of the Earth.

The tidal stretching and compression of the Earth discussed above, is often put forward as the direct cause of the tides in our oceans, but it is quite misleading, as will soon become clear. When the Sun and the Moon are lined up perfectly, the magnitude of the tidal acceleration is only about 175 nano-g on each side of the Earth.*

*175 nano-g is 175 thousand-millionth's of the normal Earth gravity, or $1.7 \times 10^{-6}\ m/s^2$.

This does stretch the Earth's radius by some tens of centimeters, but it does not cause a rise in the sea level relative to the land—the sea level and the adjacent land directly in line with the tidal vector 'rises' by approximately the same amount. However, at points on the earth's surface where both the compression and stretching effects are operating, there are components working along the surface of the Earth, as shown in figure 9.2. See, e.g., reference [Tides].

The maximum horizontal component is about 87 nano-g and occurs at points ± 45 degrees (and ± 135 degrees) from the tidal vector. If the Earth was not rotating relative to the tidal vector, this 'tractive' acceleration would eventually cause some water to flow towards points directly in line with the

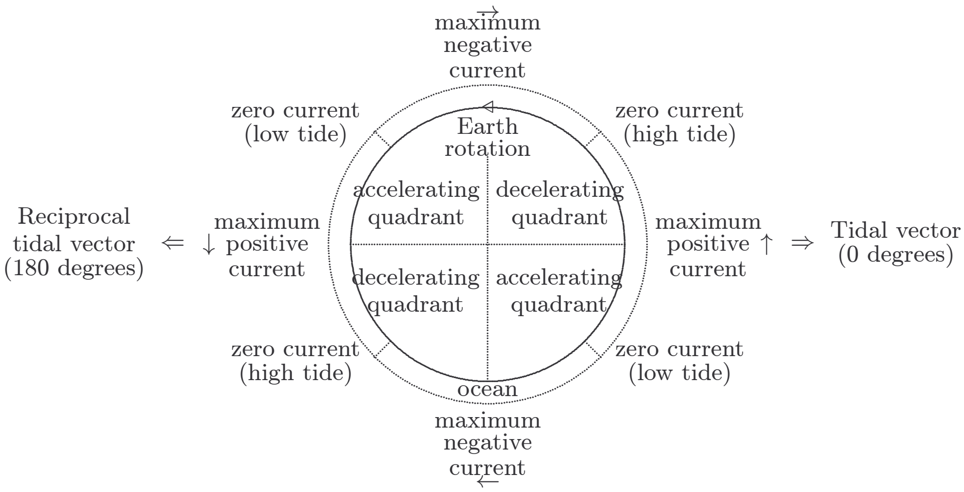

Figure 9.3: The formation of tidal currents along the equator of a hypothetical uniform, continent free ocean, as viewed from above the north pole. Tractive tidal forces alternately accelerate and decelerate water in the four quadrants, as shown. Positive currents are taken as water rotating faster than Earth's crust, flowing from west to east. The contiguous equatorial currents are ignored.

tidal vector.

The water would have heaped up a bit and at the same time caused a withdrawal of water at points 90 degrees from the tidal vector. The normal 1g gravity towards the centre of the Earth will tend to level the water and a stable equilibrium state would have been reached, with no movement of the water.

The Earth is however rotating at about 14.5 degrees per hour relative to the tidal vector and drags the water of the oceans with it. One can say that the water at the equator moves at an average speed of over 1600 km/h relative to the tidal vector.

This movement prevents a stable, equilibrium condition to be reached. As shown in figure 9.3, for the quadrants directly to the west of the tidal vector line, the tractive force causes a small acceleration of the water relative to the tidal vector, and in the eastwards quadrants, an equal deceleration.

As the Earth rotates relative to the tidal vector, each molecule of water in the open ocean along the equator will experience about 6 hours of this tiny acceleration of 87 nano-g maximum or 56 nano-g average, followed by 6 hours of equal deceleration.

The maximum positive flow, relative to the crust will occur near the two positions in line with the tidal vector. Likewise, the maximum negative flow will occur near the two points 90 degrees to the east and west of the tidal vector.

In the open oceans near the equator, it creates what is called a reversing tidal current. At other latitudes, the Coriolis force will tend to convert a reversing current into a rotating tidal current. The relative phases of the

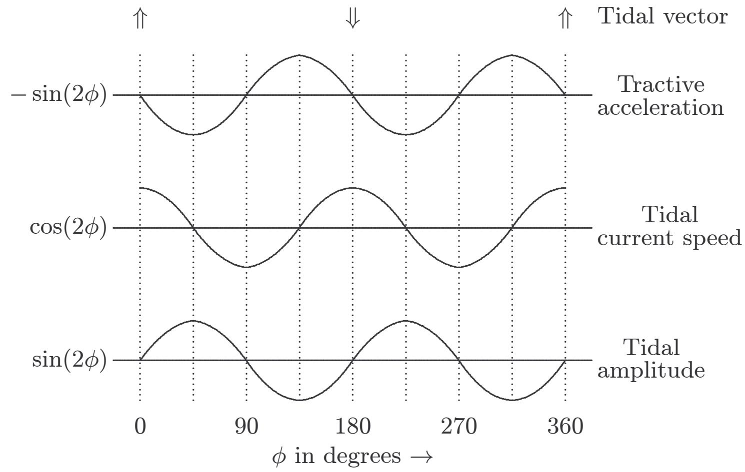

Figure 9.4: The phase relationships between the tractive acceleration, tidal current speed and tidal amplitude. Note that the peak tidal amplitude lags the tidal vector by 45 degrees and is precisely out of phase with the tractive acceleration. The amplitudes of the 'waves' are not to scale and serve only to show the phase relationships.

tidal currents are shown in figure 9.4.

To understand the phase relationships, consider water flowing through a uniform duct. If the in-flowing speed of water into any section of the duct is larger than the out-flowing speed from that section, the water level in the section must rise. This situation arises when the water is being decelerated over the length of the section.

The approximate speed of the tidal flow in open ocean is obtained by integrating the tractive acceleration, which has the general form $a_{tr} = -k\sin(2\phi)$, where ϕ is the angle measured from the tidal vector and k some positive constant. The velocity of the tidal current is then

$$v_{tc} = -k\int \sin(2\phi)\ d\phi = \cos(2\phi) + \text{ constant.}$$

As stated above, the amount of heaping of the water (the tidal amplitude) is proportional to the velocity differential between inflow and outflow of a section. The differential of $\cos(2\phi)$ is $\sin(2\phi)$, which is proportional to $-a_{tr}$, so the tidal amplitude is precisely out of phase with the tractive acceleration.

In the open (and deep) ocean, the reversing or rotating tidal currents will cause only minor high and low tides, calculated to be in the order of a few tens of centimeters. The speed of the tidal currents is extremely small.

An acceleration of 56 nano-g applied to a free moving object for 6 hours will result in a speed change of only 42 metres per hour. The reversing tidal currents in the open ocean will therefore achieve maximum speeds of less than 20 metres per hour in each direction. *So how on earth do these slow moving currents cause the coastal tides?*

The secret lurks (partially) in the considerable depth of the oceans, averaging at more than 3 km. Since the tractive tidal acceleration works about equally at all depths, the volume of slow moving water is very large and so will be the kinetic energy locked up in the movement.

If the tidal current would hit a large continent head-on, then most of that kinetic energy will be converted into potential energy by a temporary rise in the level of the coastal waters. Depending upon the topography of the coast, the heaping of water will in general tend to increase for as long as there is a nett inflow of water towards the coast. The topography may also cause the in-flowing current to be considerably larger than the mean current in the deep sea.

When the main tidal current starts to reverse under the influence of the tractive force, the elevated coastal waters will flow out to the deep sea again, where the water level is lower. This outflow is mainly driven by normal Earth gravity that tends to level the water again, assisted just a little bit by the tractive tidal force, which is near maximum at the time of high water. The water flowing away from the coastline will have some momentum and a small overshoot is possible, which added to the tractive force, causes the low tide.

When the normal Earth gravity and the tractive force are moving water in the same direction, we have the possibility of a resonating system. Whenever the natural resonance* period of the mass of water between the continents (or shallower sections) happens to be close to the period of the periodically reversing (or rotating) tidal current, we can have a sustained oscillation that causes higher than average tidal currents and with that, tidal amplitudes of a few meters above mean sea level along the coastline.

*The resonance period of an ideal rectangular water basin is $T = 2L/\sqrt{gd}$, where L is the length of the basin, g the local gravitational acceleration and d the depth of the water.

The main period of the tidal currents will average at 12.42 hours, the so called semi-diurnal period, meaning roughly twice a day. This period is of the same order of magnitude as the resonant periods possible in the many parts of the oceans of the world. We can do a rough check by using an ideal square basin as a guide. Taking a water depth of roughly the average depth of the oceans, some 3500 metres, we get a resonant period of 12 hours for a basin of 4000 km long.

The resonating areas are not limited to only the basins formed by the continents, but may also be the result of basins formed by very deep water broken up by oceanic ridges. In essence, a whole system of resonant currents can build up in the open oceans. If the natural resonant period of a basin is not near one of the main tidal periods, there will still be times when the tractive currents tend to coincide with the resonant currents, causing larger than average currents.

There will then also be times when the two currents oppose each other and smaller than average currents will result. The real basins are obviously very irregular and the resonant current systems will also overlap in places, making a complete analytical treatment almost impossible.

However, for any specific locality, the complex tidal current system will be periodic in nature and the commonly used practical approach is a quasi-empirical one, known as *harmonic analysis and prediction*. Up to 37 harmonic constituents are added together, each with it's own frequency, phase and amplitude.

The frequency, phase and amplitude values for each constituent at a specific location are obtained by analyzing observed values for current speeds and tidal heights over extended periods of time. The 37 constituents include all astronomical effects, like non-circular orbits of the Moon and the Earth, short and long term changes in declination and also the so called *shallow water effects* for the specific place on Earth.

One problem in obtaining the harmonic constituents is that the full astronomical cycle of the Moon relative to the Earth lasts for 18.6 years. This is the time that elapses before the Moon crosses the equator again at precisely the same angle and place. Tidal specialists like to have a sample of at least that long.

Once they have extracted all the constituents for a specific place, remarkably accurate predictions of current strengths, tidal heights and times can be made for many years into the future. There are however non-cyclic effects caused by high winds, low and high pressure systems and other effects of the global weather that have to be taken into account on a sporadic basis.

The simplistic theory of an 'Earth circulating wave' of water in the oceans cannot be used to directly predict the coastal tides at any location. In principle, it is not even a correct interpretation of the cause of the coastal tides.

It may be argued that the small heaping effect that lags the tidal vector by some 45 degrees does contribute to the coastal tides. This heaping can however only occur to any significant degree in the relatively deep water of the open oceans. At the coast, it is the tidal currents that contributes significantly.

Another simplistic theory which is suspect, is that the Moon's drifting away from Earth at around 3.8 cm per year, is due to the friction caused by the tides in the ocean. As we have seen, the tidal currents are reversing or rotating and friction effects will largely cancel out—there are roughly an equal amount of tidal currents hitting continents in easterly and westerly directions.

The more reasonable theory for the Moon's orbital distance to increase is related to the tidal stretch and squeeze of the Earth as a whole, causing the crust to bulge out and be squeezed inwards. This effect is circulating continuously from east to west as the Earth rotates, causing some loss of angular momentum of Earth's rotation. This loss is transferred to the Moon in the form of additional orbital momentum, because the total angular momentum of the Earth-Moon system must be maintained.

The additional orbital momentum causes the Moon to continuously take up an ever so slightly larger orbit. The physical mechanism of transfer of angular momentum from the Earth to the Moon's orbit stems from the fact that there is a slight angular lag in the bulges on Earth's surface relative to the rotating tidal vector (as viewed from Earth). This is because it takes some time for the bulges to form and to subside again.

From the Moon's perspective however, the bulges have a 'lead angle', causing the gravitational force to point slightly ahead of the common centre of gravity of the Earth-Moon system. This offset creates a small force component in the direction of the Moon's orbit, causing it to continuously pick up a tiny amount of additional orbital velocity.

The Moon also suffers tidal forces due to Earth's gravity, and they are in the order of 22 times* larger than the Moon's tidal effect on the Earth. This

> *The Earth is about 81 times more massive than the Moon, but then the Moon's diameter is about 27% of Earth's and the tidal force on the Moon is proportional to $M_{Eearth} \times d_{Moon}$.

caused large frictions in the originally faster spinning Moon and is the reason why the Moon now always shows more or less the same face to us. The Moon's rotation period has been decreased by the friction until it equaled the average orbital period of the Moon around the Earth.

Earth's rotational speed has also been decreased by tidal gravity. It causes in the order of 1.7ms in period shift per century. Stated differently, if atomic clocks and mean solar time were running precisely in step 100 years ago, then today our atomic clocks would run fast by 1.7ms every day.

This tiny amount has accumulated over time and today our atomic clocks tend to get ahead of mean solar time by about a second over an eighteen month period, on average. This is the main reason for the frequent "leap-seconds" that we experience, where we effectively set our clocks back by one second.

There are other random (unpredictable) factors caused by crustal and inner movements in the structure of Earth, that also affect Earth's rotation period. The 'scary' part of these factors is that we cannot, even in principle, predict future solar time precisely!

9.2 Relativistic tidal gravity

Relativity views tidal gravity as the result of curved spacetime. If spacetime is curved between two points, geodesics in spacetime either converges or diverges, depending on the direction of curvature.

If we take four free particles as representing a free falling object (figure 9.5), then the particle nearest to the mass has a time arrow that curves more sharply towards the mass than does the time arrow of the particle furthest from the mass, so the two particles diverge. In fact they will accelerate away from each other. The two particles perpendicular to the radial have time