Chapter 16

Measuring the shape of expansion

a wrap-up
of
'engineering cosmology'

To cosmologists, the shape of the expansion curve is all important. If they know it accurately, they will know, amongst other things, the age of the universe. They would know the distance to objects with a given redshift pretty accurately. One can understand why cosmologists wants to know the geometry of the universe.

16.1 The observables

The distance to remote objects in the universe is not directly measurable. The only direct distance related observables are the redshift and the apparent luminosity of remote objects. The relationship between these two observables and distance is however not all that clear cut.

As far as the redshift is concerned, there is a possibility that part of the redshift of distant can be attributed to 'non-cosmological' effects. This means that the redshift may not be a simple function of how much the universe has expanded since the observed radiation left the distant object. The luminosity of very distant objects may be altered by partially obscuring dust between us and the object that makes it appear dimmer than it should be.

However, by looking at objects like galaxies and supernovae near and far,* astronomers are today pretty sure that they have the tools to make long range distance measurements with errors of less than 20%. In engineering

*A supernova is essentially a massive star that ran out of fuel, causing it's core to collapse, creating a shock wave that blows the star apart.

terms, this is pretty coarse, but if that is the best we can do, we live with it.

In order to use the redshift to measure distance, one needs accurate values for the Hubble constant H_0 and the shaping parameters of the expansion curve, i.e., Ω_m and Ω_v. The latest values (at the time of writing) for these parameters came from the Wilkinson Microwave Anisotropy Probe (MWAP). WMAP determined that $H_0 = 71 \pm 3.5$ km/s/Mpc, with 73% vacuum energy.

16.2 What we expect to find

Figure 16.1 shows the situation for a luminous object with an observed redshift of $z = 1$, meaning that the light we observe left the object when the Hubble radius $a(t)r_H$ was half of what it is today. The two black dots, where the 'half-size' dotted horizontal line crosses the two curves, are the solutions for where the object was for the two cases: (i) the standard flat, decelerating expansion curve and (ii) the (presently) accelerating expansion curve, with a 73% vacuum energy component.

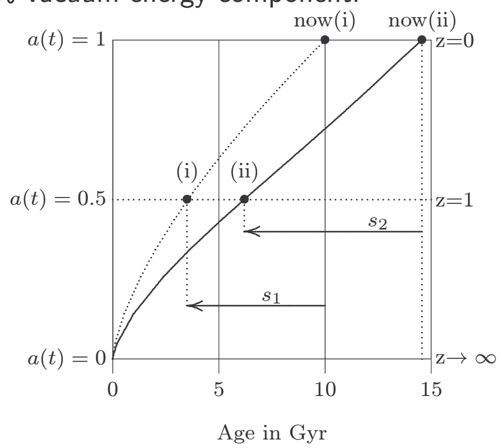

Figure 16.1: The two black dots are the positions for a celestial object at redshift $z = 1$. Position (i) is for a standard flat universe and position (ii) for a universe with accelerating expansion at the present time. The arrows (s_1) and (s_2) are the distances of the source from an observer living in the present time. The observer's position is at either 'now(i)' or 'now(ii)', depending on which curve (if either) is valid. This shows graphically that for a specific redshift, the distance to the object depends upon the shape of the expansion curve.

A Hubble constant H_0 of 71 km/s/Mpc was assumed. The two left-arrows indicate the look-back times (geometrically the same as the look-back distances) of the objects. It is immediately clear that in the case of the vacuum driven, accelerating expansion curve, the distance (s_2) to the object is larger than the distance (s_1) for the standard case. The distances can be found by numerical integration as $s_1 \approx 6$ and $s_2 = 7.7$ Gly respectively.

Now if this object had a known absolute luminosity, we could calculate the distance to the object by measuring the apparent luminosity that we observe. This would tell us which one of the two curves is the closest to the 'real thing'. If we could do that for a number of objects at different redshifts, we could in principle plot the 'real' curve and thus know the shape of the expansion.

In practice, absolute measurement of the distance to remote objects by means of apparent luminosity is fraud with uncertainties. A better approach is to measure relative distances by means of the luminosity ratio between a distant object and a nearby one—provided we know that the distant and nearby sources have the same absolute luminosities, or at least that we know the ratio between their respective luminosities.

In section 16.3, some of the measurement techniques of absolute distance (the so called 'distance ladder') is discussed in more detail. Here we will just touch upon two methods, the supernovae and the Ceiphed variable stars or *Cepheids* for short.

Cepheids are pulsating yellow giant stars radiating ten thousand times as much energy as the Sun, so they can be accurately observed at distances of more than 50 million lightyears. Further, they pulsate with a period between 3 and 50 days, depending on their luminosity—the longer the period, the more luminous.*

*Cepheid luminosity is proportional to the period raised to the power 1.3

Cepheids are also present at closer ranges, like in the Small Magellanic Cloud (SMC) and in our own Galaxy. The distances to these closer Cepheids are fairly well known from other measurements, so that an absolute distance to apparent luminosity function for Cepheids can be established to some degree of accuracy (about $\pm$ 10%).

Cepheids are not directly usable to measure the shape of the expansion curve, because they are not observable over large enough distances. They are however important in the sense that their observable range overlaps with the ranges where supernovae are reasonably abundant, like in the Virgo Cluster, some 53 million lightyears away. The Virgo cluster contains thousands of galaxies and produces enough supernovae and Cepheids that the Hubble Space Telescope can measure.

Since they are at roughly the same distance, some absolute distance to apparent maximum luminosity function can in principle be established for a specific type of supernova (the SN Ia, as will be discussed later). There is one problem though—at such distances, it is not possible to know whether the supernovae and the Cepheids are actually at the same distance from us. They appear to be part of the same cluster of galaxies, but there may be significant differences in distance.

If we combine these uncertainties with the 10% accuracy of the Cepheid function, it means using the apparent maximum brightness of SN Ia supernovae alone, we cannot presently know the absolute distance to very distant

SNe Ia to better than about 20%. We therefore have to rely mainly on relative luminosities and distances to determine the shape of the expansion curve.

The other observable parameter is the redshift of distant objects, which can be determined very accurately. However, to use redshift as a distance indicator, we need a reasonably accurate Hubble constant. Here the problem is that at the largest distance that we know with good accuracy—the Virgo cluster—we cannot measure the $cosmological$ redshift all that accurately.

At the distance of the Virgo cluster, the possible relative velocities due to gravitational and other effects are of the same order of magnitude as the pure Hubble flow (the recession velocity due to the expansion alone). This means the redshift that we measure is not purely due to the expansion of the universe.

The way astronomers attempt to overcome this problem is to measure the redshift and the apparent luminosities of SNe Ia in the Coma cluster, which is some 6 times further away than the Virgo cluster. Cepheids are not detectable at such a large distance, but the larger distance makes the influence of non-Hubble velocities less than 10% of the pure Hubble flow.

By comparing SNe Ia luminosities in Virgo and Coma, cosmologists reduce the measured redshift of Coma proportionally to obtain a 'corrected' redshift for Virgo of $z = .00395 \pm 5\%$. Using the 53 million lightyears distance to Virgo determined by Cepheids, this gives a Hubble constant of $H_0 = 980 \times .00395/.053 \approx 73$ km/s/Mpc.

Many cosmologists find this value a trifle on the high side, despite the $\pm 8\%$ error margin that is claimed. They argue that there may be systematic errors (an offset) in the process used to determine the corrected redshift of the Virgo cluster. The obtained value for H_0 does however overlap with the presently accepted range of 55 to 75 km/s/Mpc, which is about 65 km/s/Mpc $\pm$ 15%.

So it appears that when measuring large cosmological distances absolutely, using the redshift only, we can presently do no better than $\pm 15\%$. Add to this the fact that we do not know the precise expansion curve, and the precision becomes even worse.

Forewarned by the above information, let us use the simple linear Hubble law and calculate a few characteristic distances for the universe. First let us determine how far away the object of figure 16.1, with a redshift of 1 is, using the same $H_0 = 65$ that we used in the graphs.

As any radar engineer will know, the relative velocity between the target and the radar is determined by half the fractional Doppler shift (half, because the radar signal travels to the target and back again and is thus Doppler shifted twice). The fractional Doppler shift is thus twice the relative velocity, expressed as a fraction of the speed of light: $\Delta\lambda/\lambda = 2v/c$, where c is the speed of light.

In cosmology, we measure redshift only one way and therefore $\Delta\lambda/\lambda = v/c$. This is however only an approximation for relative velocities that are much,

much lower than the speed of light. Stated in another way, this is valid for fractional Doppler shifts which are much, much smaller than 1.

In the case of a larger fractional Doppler shift, the relationship of Special Relativity must be used. This gives the fractional Doppler shift as

$$\frac{\Delta\lambda}{\lambda} = \frac{1+v/c}{\sqrt{1-v^2/c^2}} - 1 = z,$$

where z is the cosmological redshift. Due to other cosmological factors, this is not quite a valid interpretation of redshift, but it is good as a first approximation.

Note that if $v \ll c$, then v^2/c^2 becomes negligible and the equation reduces to the approximation $\frac{\Delta\lambda}{\lambda} \approx v/c$, as mentioned before. To extract the velocity for a given Doppler shift out of the full relativistic equation is a messy operation, but the result can be expressed quite simply as follows:

$$v/c = \frac{(z+1)^2-1}{(z+1)^2+1}. \tag{16.1}$$

This tells us the approximate recession velocity of an object with a measured redshift of z. In cosmology, what we really want is the relationship between the redshift z and the 'look-back' distance s, which is essentially the time it took the light to have reached us from the time that it left the object.

If we assume a linear Hubble law $v/c = H_0 \times s$ (where s is the 'look-back distance'), then $s = \frac{v/c}{H_0}$, which after substituting v/c from equation 16.1 gives

$$s = \frac{(z+1)^2-1}{(z+1)^2+1} \times \frac{1}{H_0}. \tag{16.2}$$

Now we can calculate a first order distance to our galaxy at redshift $z = 1$, using a Hubble constant $H_0 = 65$ km/s/Mpc, or 65/980 per Gly in geometric units, giving

$$s = \frac{4-1}{4+1} \times \frac{978}{71} \cong 8.3 \text{ Gly}.$$

This value is a bit outside the values obtained from the graphs in figure 16.1, but not grossly so. Remember, we found 7.7 Gly for the 73% vacuum energy case, about a 7% error, showing that distance errors made by assuming a linear Hubble law out to large distances are comparable to the $\pm$ 5% error in the value of the Hubble constant itself.

The other interesting distance is the radius of the observable universe, i.e., the distance to the origin of the cosmic microwave background (CMB). The redshift of the CMB is $z \approx 1000$, making the ratio

$$\frac{(z+1)^2-1}{(z+1)^2+1} = \frac{1,002,000}{1,002,002} \cong 1$$

and the distance to the edge of the observable universe

$$s \approx 1 \times \frac{978}{71} \cong 13.8 \text{ Gly}.$$

This is just the Hubble radius for $H_0 = 71$ km/s/Mpc, which is a 'characteristic' value for the radius of the observable universe. This result is in almost perfect agreement with the 73% vacuum energy expansion curve of figure 16.1, which gave an observable universe radius of 13.7 Gly.

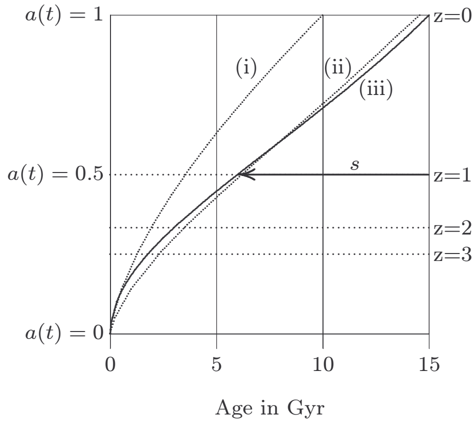

Figure 16.2: The solid curve (iii) is drawn for a linear Hubble law: $\beta = H_0 \times s$, where β is the apparent recession speed and s is the 'look-back' distance. The dotted curves are as for figure 16.1—(i) for no vacuum energy and (ii) for 70% vacuum energy. The correlation between (ii) and (iii) is remarkable, although it is probably purely coincidence, because a perfectly linear Hubble law all the way to the edge of the observable universe has no physical justification.

Figure 16.2 shows an expansion curve for a linear Hubble law, plotted together with the two curves of figure 16.1 (page 202). Amazingly, an expansion curve that produces a precisely linear Hubble law looks very much like the curve for 73% vacuum energy, complete with accelerating expansion in the second half of the history! The correlation is to all likelihood a pure coincidence because there is no physical justification for a precisely linear Hubble law right to the end of the observable universe.

Some readers may find it surprising that a linear Hubble law ($v/c = H_0 \times s$) does not translate to a linear expansion curve, but rather to the curved expansion of figure 16.2 (iii). The reason is that a linear expansion curve translates to a linear relationship between **redshift** z and **distance** s and not to a linear relationship between apparent recession velocity v and distance s (v/c and z are not the same, as shown by equation 16.1).

16.3 The distance ladder

To conclude this chapter on cosmological measurements, a brief overview of the main 'rungs' in the so called 'distance ladder' of astronomy is given. We will discuss each element, from the closest to the farthest briefly.

16.3.1 Parallax

The baseline for parallax measurement is the average radius of Earth's orbit (one astronomical unit or AU), where 1 AU ≈ 150 million km $\approx 1.6 \times 10^{-5}$ lightyears. (Engineers would like to call this 16 μl.y.!) Annual parallax must be seen as the (plus and minus) peak deviation from the mean angle of a star during one orbit of the Earth around the Sun as shown in figure 16.3.

This deviation can be measured by ground based telescopes with an accuracy of about 1.5×10^{-8} radians (0.015μrad) and by space based equipment, like *Hipparcos* (for Hight Precision Parallax Collecting Satellite), to some 5 times better, or 0.003μrad. These accuracies are not absolute, but rather relative to some very distant ("fixed") stars, where the parallax approaches zero.

To obtain a $\pm 10\%$ accuracy, the smallest parallax that can be measured is about 10 times the accuracy of the equipment, so a practical limit for ground based equipment is $0.15 \pm 10\%$ μrad and for space based equipment about $0.03 \pm 10\%$ μrad (or 3×10^{-8} radians).

Further, proper motions of stars in the transverse (angular) direction must also be accounted for, which is obtained by also measuring the positions of stars at the beginning and the end of one complete orbit of the Earth around the Sun. The true parallax is then zero and any deviation must be due to transverse motion of the star relative to Earth. Half of this deviation is then subtracted from the "6 months deviation" to obtain the true parallax.

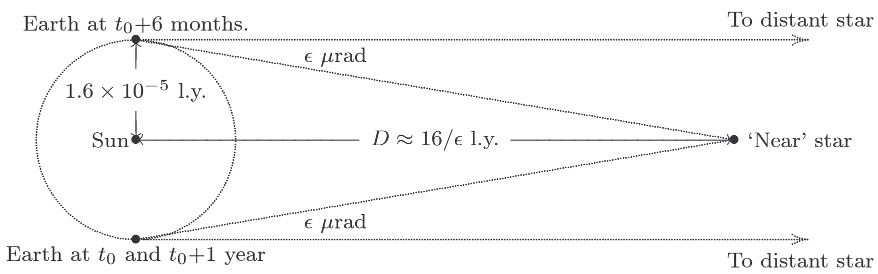

Figure 16.3: Schematical representation of parallax. In 6 months the parallax shift due to the Earth's orbit is actually 2ϵ μrad, but astronomers prefer to relate the parallax angle to the average radius of Earth's orbit, or 1 astronomical unit (AU), which equals about 150 million km, or 1.6×10^{-5} lightyears.

With a baseline of 1.6×10^{-5} lightyears (the radius of Earth's orbit) and the 10% accuracy parallax limit of $\epsilon_{min} \approx 3 \times 10^{-8}$ radians peak (Hipparcos), parallax is therefore usable out to a distance of about $\frac{1.6\times10^{-5}}{3\times10^{-8}} \approx 500$ lightyears to a $\pm 10\%$ accuracy level. At double this range the accuracy would deteriorate to some $\pm 20\%$.

Special space-based equipment of the future is likely to at least double these ranges, which we will soon see to be quite important. A very long baseline array (VLBA) project has reported some parallax measurements at 100 times the Hipparcos accuracy (Harvard-Smithsonian Centre for Astrophysics, Dec 2005), but the jury is still out on the validity. If it turns out to be correct, the range of parallax measurements might be extended to some 5,000 lightyears.*

*The problem might be that the 'right' stars may not be measurable with VLBA. See the 'standard candles' subsection below.

In engineering terms, we can say that the distance to a star is approximately $16/\epsilon$ lightyears. Here ϵ is the peak (not peak-to-peak) parallax in μrad.

Astronomers and cosmologists express parallax in *arcseconds* where 1 arcsecond $\approx 5\mu$rad. The observational limit is about $\epsilon_{min} = 0.006$ arcsecond peak.

Astronomers like to work in distance units of *parsecs*, which is simply the inverse of the parallax in arcseconds. So the equivalent distance of the furthest usable object is some $1/.006 \approx 160$ parsecs for $\pm 10\%$ accuracy. One parsec is about 3.26 lightyears.

16.3.2 Proper motion

The name 'proper motion' is a bit of a misnomer, because astronomers mean by that the angular change caused by the relative movement between Earth and the star during one year. It is measured at the same time on consecutive years and depends on the transverse movement of a star relative to Earth during the period of one year, as shown in figure 16.4.

The "real" motion of a star will usually include a radial and a transverse component, of which only the radial component is directly measurable by means of the Doppler shift of it's light spectrum. The angular motion of a star is also directly measurable, but to obtain transverse velocity from angular motion, one needs the distance—and that is the 'independent' variable that we are looking for!

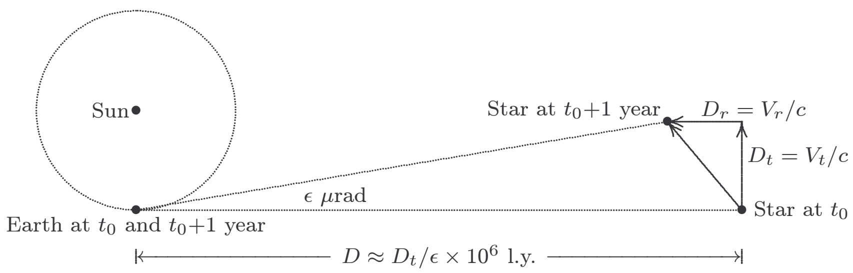

Figure 16.4: The geometry of proper motion distance measurement. V_r is the radial velocity component and V_t the transverse velocity component of a star (in km/s), D_t is how far the star has moved across out line of sight in one year and c is the speed of light (3×10^5 km/s). The factor 10^6 in the calculation of D comes from the angular movement units shown (μrad).

The transverse velocity of a star is obtained by one of two methods: the *moving cluster* or the *statistical parallax* method. In the moving cluster method, astronomers obtain the precise radial velocities of the stars in an open cluster* by means of their individual Doppler shifts.

*The nearest open cluster is the Hyades, part of the constellation Taurus, the bull. The best known one is perhaps the Pleiades or 'seven sisters', also part of Taurus.

The transverse velocities are obtained indirectly from their annual angular movement across the sky. By measuring the angular movement over many years, a "vanishing point" (as in a perspective drawing) is calculated from the combination of radial speed and angular movement.

This makes it possible to determine the approximate transverse velocity of individual stars. Once the transverse velocity V_t (relative to Earth) and the proper motion ϵ per year are known, it requires only simple geometry to calculate the distance from Earth, as shown in figure 16.4.

The second method, *statistical parallax*, does not depend on the availability of open clusters, but uses the radial velocities, as obtained by Doppler shift and the proper motion angles of a large sample of stars at various distances and directions. Sophisticated statistical methods allow the most probable transverse velocity of an individual star to be determined from it's measured radial velocity. The distance calculation then proceeds as for the moving cluster method, as discussed above.

The proper motion method improves the range of the standard parallax distance measurement, because relative to Earth, stars move much faster than Earth's orbital speed around the Sun. Add to this the fact that we have a full year's movement as a baseline and not just six months as for direct parallax.

The baseline for the annual angular displacement increases by at least as factor 10, which limits the usable range to about 5000 l.y. The relationship between transverse speed and distance is $D = D_t/\epsilon \times 10^6$ lightyears, where $D_t = V_t/c$, V_t the transverse speed in km/s, c the speed of light (3×10^5 km/s) and ϵ the annual angular movement in μrad.

Although less accurate than the direct parallax method, the importance of proper motion is that the usable range overlaps well with the distances where one of the "standard candles" of astronomy is found, as discussed next.

Figure 16.5 shows how the proper motion distances are connected to the parallax distances on the ladder. This is a 'minimal distance ladder'—there are many more 'rungs', but this serves to illustrate the principle.

16.3.3 The 'standard candles'

There are quite a few sources of radiation that have characteristic that make them good as standard sources. We will discuss only two of the most important ones here.

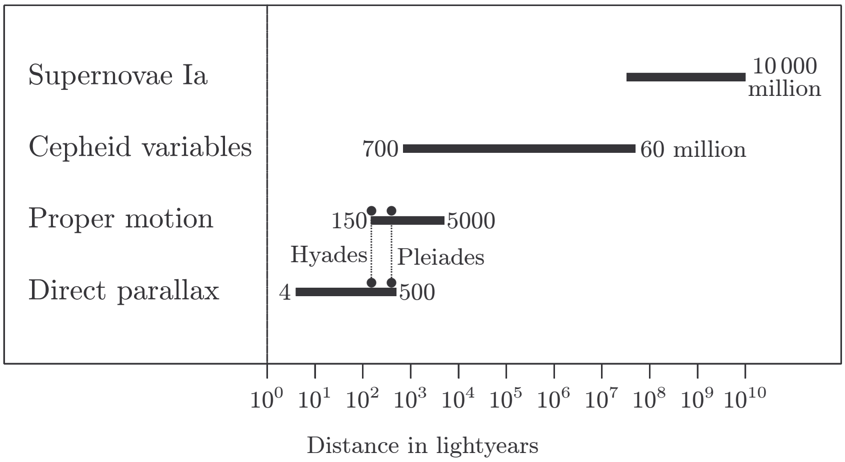

Figure 16.5: The minimal distance ladder showing only the 'rungs' for the proper motion/parallax connection.

Cepheids From a distance of 700 lightyears upwards, many so called *Cepheid variable stars* are found—the North Star (Polaris) is one of the closest known Cepheids. A typical Cepheid is a yellow supergiant star, a thousand times more radiant than our Sun.

The apparent (observed) brightness of a Cepheid varies periodically with a period that is a function of it's brightness. The brighter, the longer the period, ranging from days for a feint Cepheid to several months for the brightest ones.

Apparent brightness and distance are related by an inverse square law. So if you know the distance to a Cepheid with a specific period, you can calculate the distance to another Cepheid with that same period by comparing the brightness ratio of the two.

Further, if you know the law of absolute brightness versus the period of variation, you can in principle calculate the distance to any Cepheid from it's observed period and apparent brightness. Using Cepheids in the small Megallanic cloud (SMC), which are all at roughly the same distance from us, the law for period against brightness was found, at least in a relative sense.

Then the distance to Cepheids nearer to us (inside the Milky Way) was determined via the proper motion method. This allowed an absolute calibration for these 'standard candles' as distance measurement tools.

Due to their great brightnesses, the apparent brightness and period (and thus the distance) of Cepheids are measurable by the Hubble space telescope up to sixty million lightyears away. Figure 16.6 shows the ladder connections.

Supernovae SNe Ia The next rung in the distance ladder is the type Ia supernova, called SN Ia for short (SNe Ia in the plural). These monsters are thousands of times more luminous than the Cepheids and can be observed at distances hundreds of times further than Cepheids.

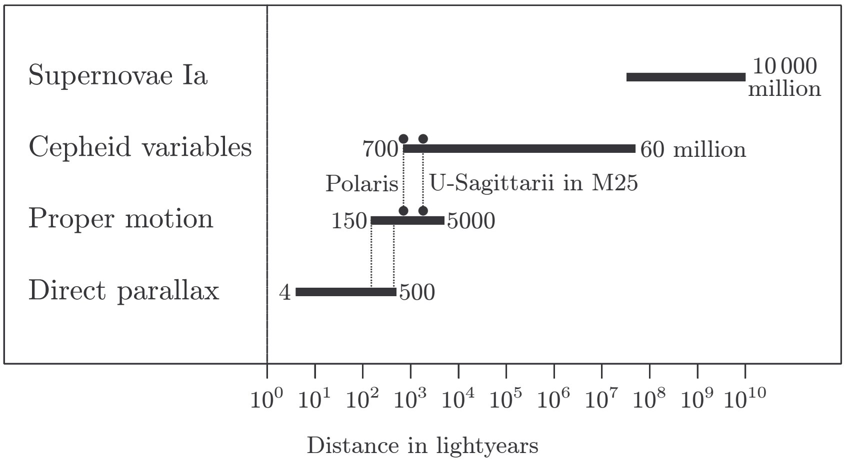

Figure 16.6: The minimal distance ladder with the Cepheid/proper motion connection added.

SNe Ia are thought to originate from white dwarf stars in binary systems that has nibbled away at the mass of it's companion star until the white dwarf's mass exceeds a critical value of about 1.4 times the mass of the Sun, causing it to go supernova and blow itself apart. As a result, all SNe Ia have a similar initial mass and thus a similar absolute luminosity.

By measuring the maximum apparent luminosity and the time constant of the flare up curve, astronomers and cosmologists can compensate for differences in mass, which influence the absolute brightness. Once one SN Ia is detected in a galaxy for which the distance is known through the Cepheid method, the type Ia supernova can be 'calibrated' and used as the 'standard candle' virtually up to the limit of the observable universe.

The accuracies of the SN 1a and Cepheid methods are relatively good, but at the time of this writing, they still depend for their 'calibration' on the proper motion method with it's uncertainties.

This gives an overall accuracy of no better than 20%. This is why the direct parallax measurement at larger distances by future space equipment is very important. If the parallax to a number of Cepheids can be measured directly, the accuracy of the whole distance ladder will improve. Figure 16.7 shows this last connecting rung in the distance ladder.

Some readers may wonder why the Hubble constant does not feature in the distance ladder. Is it not so that the Hubble constant can be used to determine the distance of objects for which the redshift is known?

The answer is yes, but only to an accuracy that is much less than what the distance ladder can provide. The present Hubble constant is known with an accuracy of about $\pm 15\%$, about the same as the distance ladder accuracy.

To use the Hubble constant as a distance measuring tool, one needs two parameters: the redshift, which can be measured pretty accurately, plus the shape of the expansion curve, which is not known with any certainty. This uncertainty causes additional distance measurement errors that are roughly

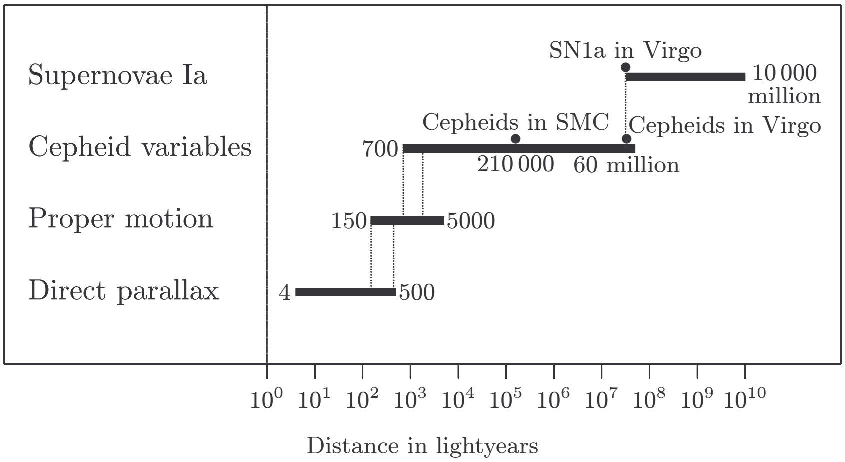

Figure 16.7: The completed (minimal) distance ladder, showing the last 'rung', the SN 1a/Cepheid connection in the Virgo cluster.

as large as the uncertainty of the Hubble constant itself.

In actual fact, any improvement in the distance ladder is used to pin down the Hubble constant more accurately and also to improve our knowledge of the shape of the expansion curve.

Cosmologists normally don't attach a distance to a specific redshift, but simply says that a newly discovered supernova or quasar is at a 'distance' of such and such a redshift.

At the start of the twenty-first century, the physical object with the largest measured redshift weighed in at about $z = 6$. Cosmologists can say with fair confidence that this object is at a distance of xxx from the edge of the observable universe.

However, to say how far this object is from us (based on light travel time), we need to know how far the edge of the observable universe is from us. In order to know this, we need the value of the Hubble constant and also the shape of the expansion curve, neither of which is known to great accuracy.