# Is the Loedel Diagram a Special Case of the Minkowski Spacetime Diagram?

**Burt Jordaan, December 9, 2007**

The main attraction of the Loedel spacetime diagram[1] is that it treats the reference frame and the first moving frame symmetrically and hence they have identical scales in geometric units. The Minkowski diagram uses different scales for the orthogonal and the non-orthogonal axes, where the time axis of the second frame is defined by an angle so that $v = \tan(\phi)$ (clockwise convention), where $\phi$ can range from $-\pi/4$ to $\pi/4$ (the light-cone). The Loedel diagram uses an angle defined by $v = \sin(\varphi)$, where $\varphi$ can range from 0 to $-\pi/2$, spanning symmetrically around the vertical through the origin, as shown in Figure 1. This gives a light-cone identical to the Minkowski diagram. One can

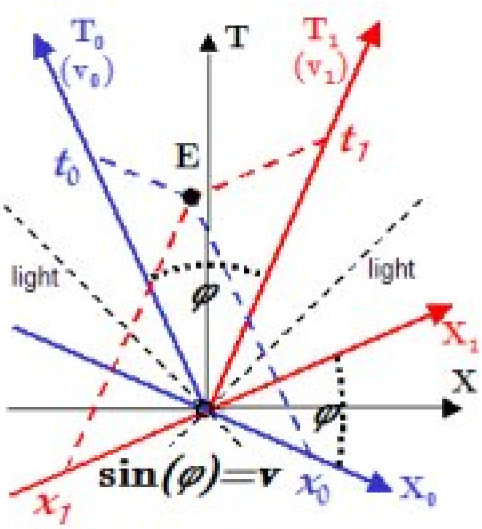


Fig: 1 Basic Loedel diagram

view the Loedel diagram as a Minkowski diagram with three inertial frames: the orthogonal axes X,T plus two frames X0,T0 and X1,T1 moving at $v_1$ and $v_0$ respectively, where $v_1 = -v_0 = \tan(\varphi/2)$ relative to X,T. The calibration of these two axis systems is identical and relate to the orthogonal axis by[2]

$$\frac{\text{scale(X0,T0)}}{\text{scale(X,T)}} = \sqrt{\frac{1+v_0^2}{1-v_0^2}}, \tag{1}$$

where ‘scale’ refers to linearly plotted length per geometric unit along the relevant axis.

It is now possible to add another inertial coordinate system, X2,T2, moving at a ‘Minkowski velocity’ $v_2$ relative to X,T and plotted at an angle defined by $v_2 = \tan(\theta)$, as in Figure 2.

The equivalent ‘Loedel velocity’ ($v_{2L}$), is measured from X0,T0 and is related

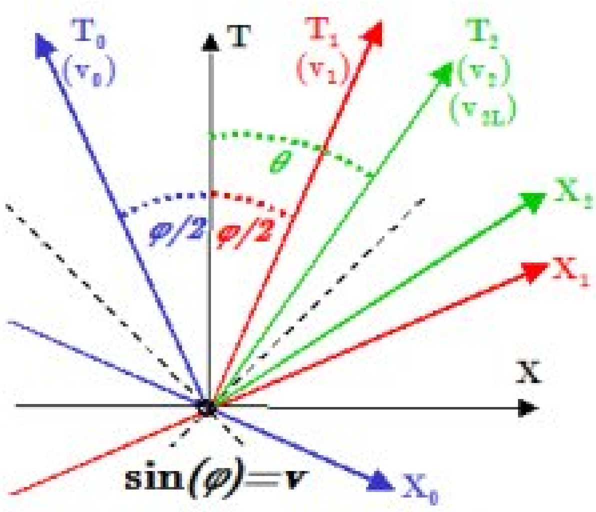

Fig: 2 Loedel diagram with 3rd frame

to $v_2$ through the relativistic subtraction:

$$v_{2L} = \left(\frac{v_2 - v_0}{1 - v_2 v_0}\right). \tag{2}$$

The scale of the X2,T2 axes is not the same as the scale of the first two, but the scale relative to the X0,T0 axes can be easily found as:

$$\frac{\text{scale(X2,T2)}}{\text{scale(X,T)}} \times \frac{\text{scale(X,T)}}{\text{scale(X0,T0)}} = \sqrt{\frac{1+v_2^2}{1-v_2^2}} \times \sqrt{\frac{1-v_0^2}{1+v_0^2}}. \tag{3}$$

The orthogonal axes X,T are now essentially 'eliminated' and we are left with a Loedel diagram with three inertial frames on it.

If $v = \sin(\varphi) = 0$, then $v_0 = v_1 = 0$ and $v_{2L}$ reduces to $v_2$, the 'Minkowski velocity'. Likewise, the relative scale of the $X_2$,$T_2$ axes then reduces to that of the Minkowski spacetime diagram. Hence the "title question": Is the Loedel diagram a special case of the Minkowski spacetime diagram? Or is it perhaps the other way round?

It is clear that the Loedel diagram wins in the simplicity stakes when there are only two reference frames involved, but the Minkowski diagram is simpler to use when additional reference frames are required. After all, additional Loedel frames need to be converted to 'Minkowskian' before being added. This probably explains why the Loedel diagram has never really caught on, except perhaps in beginners teaching.

## Sources

[1] Loedel E. Geometric Representation of Lorentz Transformation. Amer. J. Phys. 25: 327, May, 1957.

[2] Shadowitz A. Special Relativity, W. B. Saunders Company, 1968.