Chapter 10

Gravitational Waves

The 'hum, chirp and squeak'
of
our Cosmos

Gravitational waves are part of the "hot stuff" of relativity and cosmology today. It was also 'hot' for Einstein himself, way back in 1916, when he published a paper showing that his general theory of relativity predicts the existence of gravitational waves. For decades, many people considered them as of no practical value because they are so weak - just how weak we shall see later in this chapter.

There was even disagreement as to whether they were not perhaps just a quirk in the mathematics, with no real physical meaning. It was only in 1960, with the so called 'relativity revival', that other theorists provided rigorous proof (given that general relativity is right) that gravitational radiation must in fact be a physically observable phenomenon.

It is perhaps the fact that by the start of the twenty first century, gravitational waves were not yet directly observed, that makes them the hot topic that they are today.

10.1 Tidal gravity and gravitational waves

Gravitational waves are sometimes portrayed as simply the effect of oscillating tidal gravity that is felt by an object. This is a quite misleading picture. In a sense, gravitational waves are caused by tidal gravity that changes, but this is just the mechanism by which gravitational wave energy is transferred from the source to the 'fabric of spacetime'.

After being transferred to spacetime as ripples in curvature, gravitational waves propagate through space in a way analogous to electromagnetic waves—in other words as a transversal traveling wave that spreads out at the speed of light.

As we shall discuss below, the observable effects of a gravitational wave is quite different from a change in tidal gravity. If we use a fixed point on Earth as a reference, then the apparent rotations of the Sun and Moon around Earth cause periodic changes in the tidal gravity that is experienced, but this is not a gravitational wave.

It may however be argued that the rotation of the Earth and the Moon around their common centre of gravity does transfer gravitational wave energy to the vacuum. These waves may be observable at some distance from the Earth-Moon system, but they will be extremely weak.

Binary black holes as a source of gravitational waves The simplest source of gravitational waves to analyze is the case of two identical black holes in circular orbit around each other. If measured from a nearby point on the axis of rotation of the orbit (figure 10.1), a test object will experience periodically changing tidal gravity in the transverse directions.

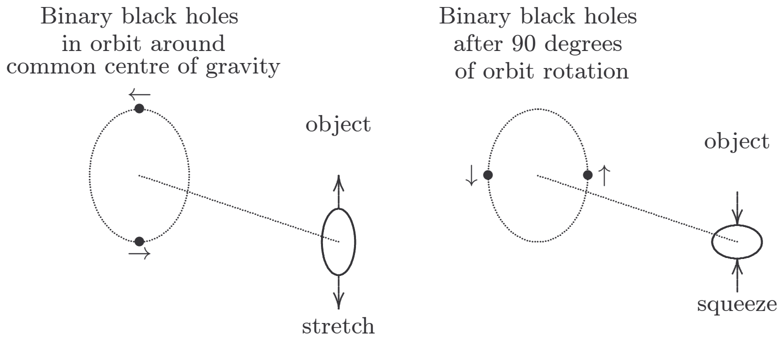

Figure 10.1: Orbiting binary black holes causing a periodic tidal stretch and squeeze on a nearby object, located on the axis of the orbit. Note that the period of the stretch and squeeze is half that of the orbital period of the black holes.

There will be a tidal stretch in the transverse directions corresponding to the orientation of the holes and a squeeze in the transverse directions 90 degrees from the stretch. The stretch and squeeze in the transverse directions will rotate with the orbit of the holes, so at each point in the reference frame, the stretch will become a squeeze and then a stretch again, with a period of half the orbital period of the two holes.

There will also be a detectable stretch in the longitudinal (or radial) direction, but that stretch will remain constant as the holes orbit around each other.

So are we measuring gravitational waves? Not quite. We essentially measure straightforward tidal gravity caused by the orbiting black holes. In pure Newtonian gravitational theory, tidal gravity would be all that there is.

As we have seen before, tidal gravity effects diminish rather rapidly with distance, because the strength is inversely proportional to the cube of the distance.* So tidal gravity is essentially a short range effect.

In General Relativity, tidal gravity is explained as curvature in spacetime. So oscillating tidal gravity must be oscillations in the curvature of spacetime. The periodic squeezing and stretching is caused by curvature that alternates from being positive to being negative in the transverse directions relative to the source.

Einstein's relativity theory demands that such oscillations in spacetime curvature will progress through space at the speed of light, as a transversal wave (similar to electromagnetic waves). Objects through which this wave passes, will alternately experience a squeeze and a stretch in directions perpendicular (or transverse) to the direction of the wave's movement.

One can also say that gravitational waves are spacetime curvature that alternates from being 'concave' to being 'convex', as shown in figure 10.2.

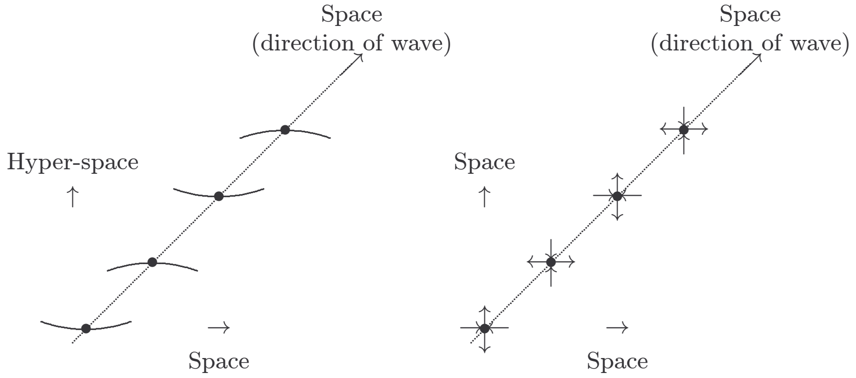

Figure 10.2: Gravitational wave principles. On the left is a space-propertime diagram, showing alternating directions of curvature of spacetime for one transverse space direction (the curvature is in the hyper-space direction). There are similar oscillations in other transverse space directions, which are not shown here. On the right, the stretch- and squeeze effects of the curvature oscillations on normal three dimensional space are shown.

One can say (loosely) that the orbiting black holes act as a transmitter of gravitational waves, while the space in their immediate vicinity works like an antenna. Relativists call the region inside one 'reduced wavelength' ($\lambda/2\pi$) from the centre of the transmitting system, the 'near zone',* which effectively acts as an antenna.

This transmitter/antenna combination generates two distinct waves, 90 degrees out of phase and polarized at 45 degrees relative to each other. The reason for the 45 degree relative polarization comes from the fact that the

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*Tidal acceleration is proportional to $\bar{M}d/r^3$, where d is the diameter of an object and r the distance from the source with mass $\bar{M}$.

*E.g., [MTW, section 36.10, figure 36.3] spacetime oscillations have a frequency of double that of the orbital frequency of the black holes, as illustrated in figure 10.3.

The two polarizations are referred to as the 'plus' waves and the 'cross' waves and indicated by + and × respectively.

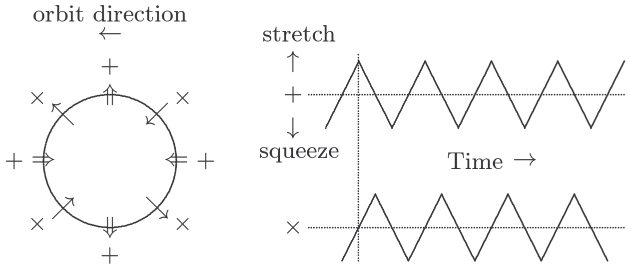

Figure 10.3: The two gravitational wave polarizations (+ and ×), showing their relative spatial orientation of 45 degrees (left) and relative phase shift of 90 degrees (right). The 45 degrees polarization results from the fact that the waves repeat themselves after half an orbit of the holes. Gravitational waves will not generally have a triangular shape, as shown here for simplicity.

The energy density of gravitational waves is proportional to the inverse of the square of the distance (i.e. $1/\mathbf{x}^2$) traveled, just like electromagnetic waves of light, radio transmissions, etc. Very importantly though, the amplitude of such waves (gravitational and electromagnetic), is proportional to the inverse of the distance traveled ($1/\mathbf{x}$), as depicted in figure 10.4.

As far as amplitude is concerned, we can say that in comparison to tidal gravity, a gravitational wave has a long range effect—it falls off relatively slowly with distance. Oscillating tidal gravity transfers orbital energy to spacetime as curvature vibrations at close range and gravitational waves then propagate this energy to large distances, where direct tidal gravity changes are utterly negligible.

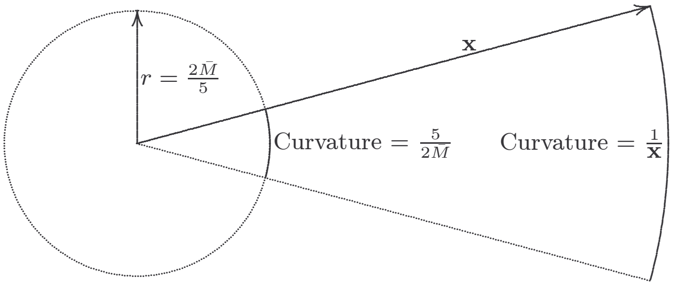

Figure 10.4: A simplistic view of how the strength of a gravitational wave is diminished by distance. The radius of the circle comes from Einstein's field equations for two coalescing black holes with total mass M. If the geometric curvature $(\frac{5}{2M})$ of the circle represents a gravitational wave strength of 1 unit, then the geometric curvature $(\frac{1}{\mathbf{x}})$ of the circle segment at distance $\mathbf{x}$ represents a strength of: $(\frac{1}{\mathbf{x}})/(\frac{5}{2M}) = \frac{2\bar{M}}{5\mathbf{x}}$ units.

The emission of gravitational waves by orbiting bodies causes a loss of orbital energy, making the two black holes to spiral inwards towards their common centre of gravity. They will eventually spiral into each other to form a single black hole.

The unified black hole will initially be very non-symmetrical - it will have two bulges protruding from it's “equator”—the plane perpendicular to it's rotation. The bulges will be moving at close to the local speed of light and will emit large amounts of energy in the form of gravitational waves.

The loss of energy will, within a short time, cause the bulges to dissappear and the black hole will become circularly symmetrical around it's equator. By that time, more than 5% of the original mass-energy of the two black holes would have been lost due to gravitational wave emissions.

The whole process is called the coalescence of two black holes and it is thought to happen often during the early life of galaxies. Because the process is relatively well understood, this type of occurrence is a prime candidate in the quest to measure gravitational waves directly.

10.2 Detection of gravitational waves

The active search for gravitational waves started in the early 1960s, when Joseph Weber started to work with his gravitational wave detectors, the so-called Weber bars. We will return to this later, but first it is worthwhile to briefly examine the indirect evidence for the existence of gravitational waves.

In 1974, Hulse and Taylor discovered the first known binary pulsar, believed to be two neutron stars orbiting each other at fairly close range. They are estimated to be about 16,000 lightyears from Earth and weigh in at about 1.4 solar masses each. The orbital period of the binary system is about 27,000 seconds, from which the average separation between the two neutron stars can be computed to be about the same as the diameter of our Sun (≈ 6.4 lightseconds).*

*The separation for circular orbits is $(\frac{\sqrt{\bar{M}}\times Period}{2\pi})^{\frac{2}{3}}$, where $\bar{M}$ is the combined mass.

This separation is not close enough to emit gravitational waves of significant amplitude and to make matters even worse, the period of the waves will be about 13,500 seconds, giving a frequency of 1/13,500 Hz, or 74μHz. So direct detection of the gravitational waves is highly unlikely.

If however, the binary pair is spiraling in towards each other due to the orbital energy lost to gravitational waves, this should be detectable as a decrease in the orbital period. The theoretical decrease is also very tiny, just 75μs per year, and thus difficult to measure.

In 1983, Taylor and colleagues refined the measurements enough to report a period decrease of $76\pm2\mu$s per year. There is no other plausible explanation for this decrease in orbital period—so the existence of gravitational waves has been verified experimentally.

10.2.1 Weber bars

In the late 1960s, Weber had designed and built a detection system that could measure gravitational wave stretching and squeezing of a solid bar of aluminum, 1 to 2 metres long and half a metre in diameter, to an accuracy of 10^{-16} metre (that is one-tenth the diameter of the nucleus of an atom).

The reason for the size of the bar is that it would have a natural resonance frequency around 1000 Hz, a frequency that should theoretically be present in some of the strongest gravitational waves. Once the bar is disturbed by the tiny gravitational wave energy at a frequency near it's natural frequency, the bar will amplify the signal slightly and Weber picks up this vibration by piezoelectric crystals glued around the middle of the bar.

By stringing a large number of crystals in series, the voltage output would be detectable if the bar is exited by a gravitational wave with an amplitude of at least 10^{-16} metre of displacement. Although Weber claimed success in the late 1960s and early 1970s, other experimenters failed to confirm the results experimentally.

It is today accepted as unlikely that Weber did pick up gravitational waves, because more in-depth theoretical research has shown that the likely strength of gravitational waves reaching Earth would be a factor hundred thousand times smaller than the sensitivity of Weber's design.

The reason for the incredibly small amplitude is the large distance at which significant gravitational waves are likely to be created. The standard argument goes something like this: supernova explosions are strong sources, but the occurrence is about one per 100 to 300 years in a typical galaxy. We are unlikely to see one near us soon.

Coalescing black holes are strong sources, but they occur mostly at the core of young galaxies. All the galaxies near us are old and the closest promising candidates are thought to be at 'cosmological distances', more than 1 billion (10^9) lightyears away.

Very near two coalescing black holes, the gravitational waves will stretch and squeeze any object by about the same amount as it's size. Relativists call this a strength of 1 unit—it amounts to 100% stretch and squeeze. At large distances the strength is approximated by two-fifths of the total mass of the holes, divided by the distance (all in geometric units), as per figure 10.4.

For coalescing black holes with a combined mass of ten of our suns ($\approx 1.5 \times 10^4$m), at a distance of 10^9 lightyears ($\approx 10^{25}$m), the strength is $\frac{2}{5} \times \frac{1.5\times10^4}{10^{25}} \approx 10^{-21}$. The strength of a specific gravitational wave is then multiplied by the dimensions of the object to determine the amplitude of the stretch or squeeze.

For Weber's original 1 to 2 metre long bars, the stretch and squeeze caused by the gravitational wave are 10^{-21} metre, or one millionth of the diameter of an atomic nucleus. The best bar detectors of today have design sensitivities in the order of 10^{-17} strength, still a factor ten thousand from the theoretical requirement. So how do the experimental physicists plan to bridge the gap?

10.2.2 Laser interferometry

The plan is to build a *laser interferometer* as a gravitational wave detector, using, instead of bars, three mirrors suspended by wires from overhead supports. The three mirrors will be arranged in a horizontal "L" shape, with each of the arms of the L several kilometres long.

By means of laser interferometry, experimentalists hope to detect the tiny changes in the length of the two arms, caused by the fact that one arm will be stretched at the same time as the other arm is shortened by a passing gravitational wave.

The USA government is investing funds into the building of a National Science Foundation facility called *LIGO*, for *Laser Interferometric Gravitational-wave Observatory* Figure 10.5 shows a schematic of the type of interferometer used in the LIGO system..*

*For a summary of the mechanics, optics and politics of LIGO, see chapter 10 of [Thorne]. Alternatively, see [Sigg] on the Internet.

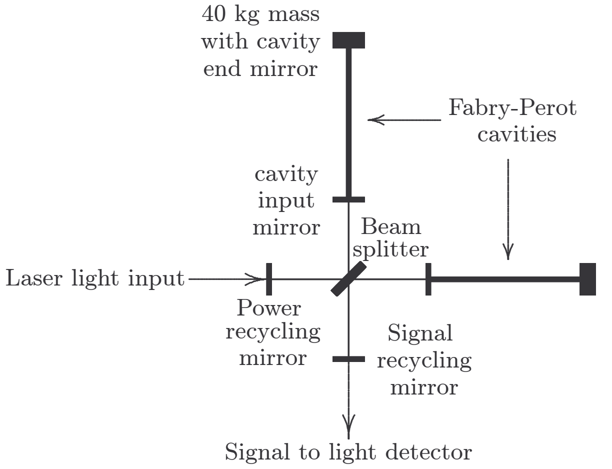

Figure 10.5: The LIGO optical paths in simplified schematic form. Properly tuned in length, the two Fabry-Perot cavities recycle and resonate the light input inside them to achieve hundreds of times the light energy of the input. The same thing happens, to a lesser degree, between the cavity input mirrors and the recycling mirrors. If the arm lengths are the same, then no light arrives at the detector, since the interference is destructive. See text for more details.

All interferometric gravitational wave detectors use variants of the Michelson interferometer, similar to the device that Michelson and Morley used in 1887 in their attempt to detect the movement of the Earth through the aether.

The basic principle is that of light passing through a beam splitter that sends half the light down one arm and half the light down the other arm. At the end of each arm, the light is reflected back to the beam splitter that recombines the light, half towards the input source and half in a direction 90 degrees from the beam splitter, where a light detector is situated.

With proper tuning of the length of the two arms, the light can be made to interfere destructively in the direction of the detector, so that no light is measured there. If however, a gravitational wave shortens the one arm and lengthens the other, (in other words, it disturbs the length tuning of the arms), the interference will not remain destructive and there will be a measurable light output going to the detector.

The LIGO apparatus uses a dual recycled Michelson interferometer with Fabry-Perot cavities, as shown schematically in figure 10.5. The Fabry-Perot cavities are tuned in length to match the wavelength of the input laser's light and the cavity input mirrors reflect most of the light returning from the cavity rear mirrors back to the rear of the cavities.

This light adds to the input light and sets up a resonating system that builds up the light intensity in the cavities to hundreds of times the input light. Some of the light does however escape through the cavity input mirrors and would have been lost if it was not for the power- and signal recycling mirrors, which reflect most of that light back into the system.

The design goal is to get a total light amplification in the system of a few thousand times. In essence, this means that the laser light is recycled thousands of times through each arm, multiplying the phase shift caused by stretching and squeezing of the arms by the same amount. This phase shift causes some light to “leak out” towards the photo-detector because the destructive interference is no longer complete.

The output light will be amplitude modulated by the gravitational wave. Detection is done much like in a standard superheterodyne radio receiver, where the carrier frequency is converted to an intermediate frequency that can be amplified and then demodulated to extract the gravitational wave signal.

A prototype laser interferometric detector with arms of 40 metres has been built at Caltech in the late 1990's to demonstrate the engineering feasibility. One detector with 2 km arms and two detectors with arms of 4 km are under construction - two at Caltech and one at MIT. One needs at least two detectors at different locations in order to eliminate local noise sources.

In late 2001, the LIGO team was busy with engineering runs in order to debug and calibrate the horrendously complex optics, control systems and noise suppression mechanisms. If all goes well, they will be able to reach the elusive sensitivity of 10^{-21} strength between 10 Hz and 10 kHz. In fact, over this frequency range, the design sensitivity of the 4 km detectors is ten times better than the minimum theoretical requirement of 10^{-21} strength.

In the centre range of 50 to 500 Hz, the hope is to achieve a sensitivity hundred times better than the minimum theoretical requirement, i.e. 10^{-23}

strength. This is the frequency range where coalescing neutron stars should be transmitting strongly.

The most important effect of a 100-fold increase in sensitivity is that the expected rate of detection of strength 10^{-21} sources becomes a million times better. The distance at which sources become detectable increases only 100-fold, but the volume of space where detectable sources may lie increases by $100^3 = 10^6$.

The planned schedule of LIGO is to start scientific runs in 2003 with the 2 km detector and soon after that with the other two. There are also laser interferometric detectors being constructed in Germany, Italy and Japan. Once all the planned detectors are in operation, it gives a relatively long baseline system for pinpointing the direction in space from which the waves are coming with reasonable accuracy.

10.2.3 Cosmic music

The bandwidth of LIGO (10 Hz to 10 kHz) is just about the whole audio frequency spectrum, meaning that if you plug the output from the detector into audio equipment, you can listen to the gravitational waves directly - a sort of cosmic music. On the LIGO web pages, one can find simulations of gravitational waves that you can listen to.

For our previous example of two coalescing black holes with a combined mass of ten Suns, the audio will sound like a “chirp”, building up in amplitude and frequency to reach a crescendo near 1 kHz. The chirp effect is caused by the inspiral phase, when the period of the orbit decreases and the orbital velocity increases - so both the amplitude and the frequency of the waves increase.

Just before the two holes merge, the orbit velocity approaches the speed of light, causing the crescendo at maximum amplitude. After the crescendo it ‘rings down’ in amplitude at a constant frequency of about 2 kHz.*

The merged, but still deformed hole will usually spin at close to the maximum possible rate and produces waves at a constant frequency of roughly twice the crescendo frequency.* As the hole smooths itself out, the gravitational wave amplitude rapidly diminishes to zero.

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*The crescendo frequency is about 10 kHz divided by the number of solar masses represented by the combined mass of the two black holes.

*The maximum possible spin rate is that of the *extreme Kerr black hole*, as discussed in chapter 6.

10.2.4 What lies ahead

Further into the future, there are plans to put a laser interferometer into space. It is called the LISA project, for Laser Interferometer Space Antenna, with the intention to put 3 satellites in solar orbit forming a large equilateral triangle.

The main objective of the LISA mission is to observe very low frequency (10^{-4} Hz to 10^{-1} Hz) gravitational waves from galactic and extra-galactic binary systems and gravitational waves generated in the vicinity of the very massive black holes believed to occupy the centres of many galaxies. These monster black holes can "eat" whole stars and the movement of such large masses generates strong gravitational waves.

The frequency is however very low because the event horizons of such black holes are so large that the orbital period of the in-falling stars, even approaching the speed of light, is relatively long.

For a non-rotating black hole with mass M, light will take $6\pi M$ seconds to orbit the hole, giving for a hole weighing in at a million Suns, a minimum period of $6\pi \times 5 \times 10^{-6} \times 10^{6} \approx 100$ seconds, equivalent to a maximum gravitational wave frequency of 10^{-2} Hz. Only a space based system can possibly to detect the occurences of super-massive black holes swallowing stars.

10.3 The purpose of it all

All the effort and expense for the projects mentioned above, would be utterly senseless if the aim was just to detect gravitational waves for the sake of detecting it. Gravitational waves promise to open a new branch of astronomy - it is after all why LIGO is called an observatory!

Despite the immense difficulties involved in the detection and recording of gravitational waves, if successful, they will provide a new and different view of astrophysical processes largely hidden from electromagnetic astronomy, such as super-massive black holes in the centres of galaxies gobbling up matter and the inner dynamics of supernova and neutron star cores.

It may eventually tell us just how right Einstein was - or when and where his general theory of relativity starts to break down. Further, by measuring low frequency background gravitational wave signals from the very early universe, it may help to discriminate between various cosmological models.