Chapter 12

Introduction to Cosmology

how
engineers might view
the cosmos

Cosmology is the study of universe at large. It is an attempt to make physical sense of the material cosmos, where it came from and where it is heading.

This text is written for the engineers and the likes of them. One could ask the question: why would engineers be interested in cosmology? For one thing, the cosmos can be viewed as one extremely large machine that follows the laws of physics in it's operation. Machines of all sorts, and especially the design of such machines, are the domain of engineers.

Further, the 'cosmic machine' fulfils the very practical purpose that it provides a home for mankind to live in. So it is natural that we would like to know how the machine works. The problem is that it is somewhat as if we are elementary particle-sized beings, living on one of the spinning ball bearings in this gigantic machine.

With the little that we can observe from where we live, we try to reverse engineer the machine—at least the cosmologists do, if they will forgive me for the engineering approach. In this approach, the objective is to reconstruct the engineering specifications, algorithms, parts lists and raw material requirements of this complex machine.

12.1 Cosmic design

Taking their cues from the cosmologists, engineers may possibly view the cosmological 'machine' (from inside out, or 'bottom-up') as follows. Our 'ball', the Earth, is an elementary part of a basic component (the bearing), which we call the solar system. Many such basic components make up a 'sub-assembly' called the Milky Way galaxy.

We know that there are other similar galaxies that are gravitationally coupled to our Galaxy, amongst them the Andromeda galaxy and the two Magellanic clouds. So the 'sub-assemblies' are integrated into a larger 'assembly' of galaxies that is called the Local Group.

There are other groups of galaxies, some much larger than our Local Group, which are called clusters due to their size. The Local Group can be viewed as a 'mini cluster'. The groups and clusters are integrated into a 'sub-system' called a supercluster.

Our Local Group is part of the Virgo supercluster, so called because the Virgo cluster is an extremely large collection of galaxies that we observe in the direction of the Virgo constellation of stars. There are many such superclusters that we can observe and they are all integrated into a larger 'system'—the observable universe.

The integration seems to be via connecting elements made of filaments or sheets of galaxies, leaving huge, bubble-like volumes of empty space between them. The empty 'bubbles' are called cosmic voids.

At present it does not seem as if there are larger scale substructures in the cosmic 'machine', so for the purposes of this book (and for the benefit of the system engineers), this is our 'system'.

This is a gross oversimplification of the structure of the real cosmos, but it serves to illustrate the idea. Figure 12.1 depicts the simplified hierarchy and some characteristic sizes of the system.

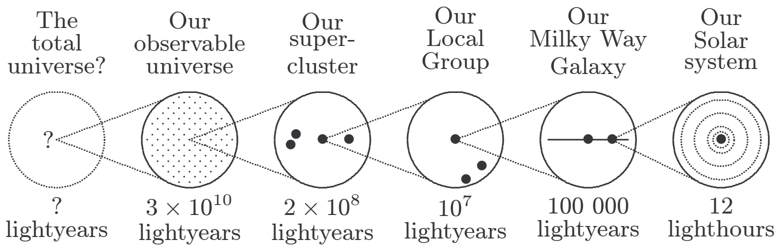

Figure 12.1: A simple hierarchical view of our place in the cosmos and some characteristic sizes. The leftmost circle represents the entire universe, of unknown size. The other circles represent small portions of the circle on it's left. Only major or important components are shown in each circle.

A supercluster is in diameter some 10^{11} times as large as our solar system. Further, a typical supercluster has a diameter of almost 1% of the observable universe. This gives some idea of how large a supercluster actually is.

Everything up to groups and clusters of galaxies seems to be gravitationally bound in the sense that their components seem to be orbiting the centre of gravity of the structure.

Superclusters do not appear to be gravitationally bound in this sense, due to the large distances between the clusters and the fact that gravitational influence diminishes with the square of distance. For example, the distance between us and the Virgo cluster is presently some 60 million lightyears. The Virgo cluster, being so massive, does gravitationally influence other clusters in the supercluster, but not in the sense of creating bound orbits.

The approximate diameters and distances that were quoted above are all in units of light travel time, as is implied by the units light years. We will later see that light travel time does not necessarily mean the physical distance between objects.

12.2 The expanding universe

The expansion of the universe is normally visualized by means of a balloon that is being inflated. A better and perhaps, by modern knowledge, a more correct visualization is that of an infinitely large lattice of rods, normally referred to as *Escher's infinite lattice* [Gribbin, page 43]. Figure 12.2 shows just one (cubic) element of such a lattice, consisting of nodes (the black dots) and connecting rods.

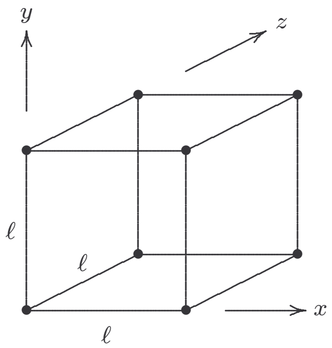

Figure 12.2: One element of Escher's lattice, with rod length ℓ. Repeat this element indefinitely in all directions and we have **Escher's infinite lattice**.

Imagine this element being duplicated indefinitely in all directions and we have Escher's infinite lattice. Let the rods represent space and the nodes superclusters, with the length of all rods being ℓ units at the present time. If in every time increment, say Δt, all rods were to lengthen by some constant amount, say $\Delta\ell$ units, the recession speed between any two adjacent nodes will be

$$v_r = \Delta\ell/\Delta t$$

units (see figure 12.3). The recession speed between any node and the second node from it will be

$$v_r = 2\Delta\ell/\Delta t$$

units, because between them there are two rods that each lengthens by $\Delta\ell$ units in time Δt. In general we can say that the recession speed between any two nodes is

$$v_r = n\Delta\ell/\Delta t = nk,$$

where n is the number of rods between the nodes and $k = \Delta\ell/\Delta t$, which is constant, as defined above. This is essentially *Hubble's law* for the recession velocity of galactic clusters, discovered by Edwin Hubble in the early 1920's.

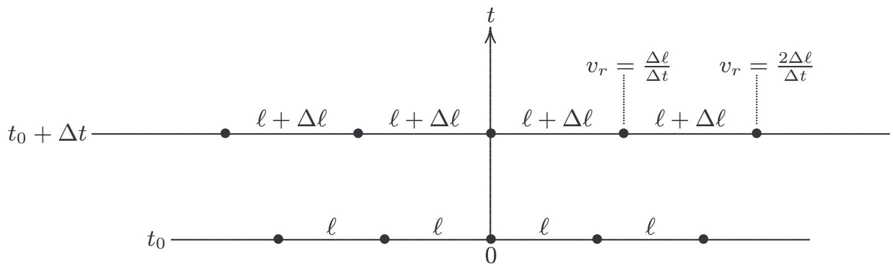

Figure 12.3: Two views of a one dimensional portion of Escher's infinite lattice, one for time t_0 and one for time $t_0 + \Delta t$. In the time Δt, distances between nodes stretch from ℓ to $\ell + \Delta\ell$. The recession speed of the first node form the origin is $v_r = \frac{\Delta\ell}{\Delta t} = k$ and the speed of the second node from the origin is $v_r = \frac{2\Delta\ell}{\Delta t} = 2k$.

The law states that the apparent recession velocity of a cluster, as measured by it's cosmological redshift is directly proportional to it's distance from us. The constant of proportionality has been aptly named the *Hubble constant*, H, i.e.,

$$v_r = sH,$$

where s is the distance to the cluster as measured by light travel time. Since it is possible that H may change over time, the present value of the Hubble constant is usually denoted H_0 and is expressed in km/s per Megaparsec (Mpc).

One Mpc is about 3.3 million lightyears. By the turn of the century, the measured value of H_0 has converged to around 64 km/s/Mpc. A value of about 73 km/s/Mpc has lately appeared to be the 'best buy', but it may still change in time to come.

So we will stick, for the time being, to the older value. This means that a galaxy at a distance of 1 Mpc (or 3.3 million lightyears) should appear to recede from us at a radial speed of 64 km/s.

Now this is one enormous speed if reckoned by terrestial standards. Unfortunately for astronomers, this is a rather low velocity by astronomical standards. Earth is moving at about 30 km/s in it's orbit around the Sun. The Sun itself is moving at over 200 km/s around the centre of the Milky Way.

Now add in the Milky Way's orbit around the centre of gravity of the Local Group and the fact that the whole Local Group seems to be moving at around 600 km/s. This movement is in a direction towards a presumed conglomeration of superclusters called the 'Great Attractor'.

You can imagine the problem. To extract the *pure Hubble flow* (the 64 km/s in the example) from the so called *peculiar motion* of galaxies of up to 600 km/s, is a difficult task. It is only for galaxies or clusters at distances of a few hundred million lightyears from us that the Hubble flow dominates the observed velocity.

For example, a cluster at 330 million lightyears will have a Hubble flow of about 6400 km/s, which is some ten times that of the peculiar velocities. How distances of hundreds of millions of lightyears are measured will be discussed in a later chapter.

Our constant k is essentially the same thing as H, although we use a 'quantized' distance in the form of the number of 'Escher rods' between us and the cluster. Make the rods short enough and k is indistinguishable from H, at least for the present state of expansion of the universe.

If we arbitrarily assume that at present the Escher rods are one lightyear long and we express velocity as a fraction of the speed of light (so that velocity is a dimensionless quantity and the speed of light is unity), k has the units lightyear^{-1}.

The conversion from H to k involves changing km/s to a fraction of light speed and converting Mpc to lightyears. The result is

$$k \approx H \times 10^{-12} \text{ lightyear}^{-1}.$$

A very interesting (but quite naive) question to ask at this point is: if the universe works like Escher's infinite lattice, how far can an object be from us before it's recession velocity reaches the speed of light? It is just the inverse of k, i.e., $10^{12}/64 \approx 1.6 \times 10^{10}$ lightyears.*

*Recall that $v_r = nk$, so that $n = v_r/k$. Here $v_r = 1$ and n is the number of rods of length 1 lightyear each.

This roughly correlates with the radius of the observable universe—we can presumably not observe anything that, due to expansion of the universe, is receding from us at a speed higher than the speed of light. We will later see that this statement is a bit misleading in an expanding universe.

If Escher's infinite lattice always followed the above law in the past, then there must have been a time when the length of all the rods between the nodes must have been zero. How long ago would that have been? We simply calculate how long it would take a rod, starting at zero length, to reach it's present length (which is one lightyear according to our units).*

*Recall that $k = \Delta\ell/\Delta t$, where here $\Delta\ell = 1$ lightyear.

The answer is again

$$t = 1/k = 10^{12}/64 \approx 1.6 \times 10^{10} \text{ years},$$

the same numerical value as the one that we obtained above for the radius of the observable universe. It makes some sense that one can observe things only as far as light has had time to travel since the 'beginning'.

With rod lengths close to zero, the 'beginning' must have been a place approaching infinite density, but possibly still of infinite size—an infinite number of rods of infinitesimal length still add up to an infinite size!

On the other hand, our observable universe is of finite size and at the beginning this 'patch' of the universe must have been very close to a single point. Do we know where that single point is (or was)? Yes, it is right here where we are, because our present place must have been at that point—just like every other place in our observable universe must have been at that point. The 'single point' has become the observable universe.

The simplistic 'Escher model' does not represent the present thinking about the dynamics of the universe. As we shall see in chapter 16, it comes very close (at least for the present epoch of expansion) to one of the 'latest and greatest' theoretical schemes that fits observational evidence pretty well.

12.3 The cosmologist's approach

In this introduction, only some of the principles and symbology that cosmologists use in their technical treatment of cosmic models will be given. A more detailed treatment will follow in later chapters.

Firstly, cosmologists prefers to work with what they call *comoving coordinates*, a fancy name for a rather simple concept. They are effectively just counting the rods in the 'Escher model'.* The number of rods between two

*There is no official 'Escher model' of the universe—it is just a convenient reference.

any two nodes obviously remain constant, no matter how much the rods expand, or shrink for that matter.

This is equivalent to the comoving distance between clusters and is denoted by the symbol r,* which is obviously not the same as the *proper distance*

*Comoving distance is also denoted by the symbol χ. See table 12.1 below.

between the two clusters. Proper distance is measured by a non-stretching ruler, which can be done by measuring the time an electromagnetic signal takes to propagate the distance.

The proper distance increases in an expanding universe and cosmologists obtain it by multiplying the comoving distance (r) by a time varying *expansion factor*, denoted by $a(t)$, i.e., a as a function of time. The proper distance is thus

$$\ell = a(t)r,$$

where the expansion factor $a(t)$ is chosen so that at present, $a(t_0) = 1$.

When the rods of Escher was half their present size, then $a(t) = 0.5$. As an example, a galaxy that today has a comoving distance of $r = 100$ million lightyears will have a proper distance $a(t_0)r = 100$ million lightyears, because $a(t_0) = 1$. A long time ago, when the expansion factor was $a(t) = 0.5$, the comoving distance of the galaxy was still 100 million lightyears, but it's proper distance was $a(t)r = 50$ million lightyears.

Cosmologists sometimes drop the t in $a(t)$, and just refer to the time varying expansion parameter as a. The expansion factor $a(t)$ is of great importance in cosmology, because it tells us how the expansion has changed over time.

The 'Escher model' is essentially a flat, Euclidean space model. Space may not be completely flat, but may have curvature. Curvature was treated thoroughly in the first part of this book, on relativity. We will 'borrow' from general relativity a hypothetical three dimensional 'hyperspace' and then mathematically embed normal three dimensional space is into this hyperspace domain.

Now normal space can curve into the extra (hyperspace) dimensions, just like the two-dimensional surface of the Earth curves into the third space dimension. With this postulate, one can construct an Escher lattice, so that it has curvature, yet with all rods having the same length. This is impossible in normal three dimensional space.

It is also impossible to visualize such a lattice, let alone sketch one. We are forced to drop one space dimension and consider only two dimensions of space as all there is. The third dimension can now be 'viewed' as one dimension of hyperspace.

'Viewed' is actually a bad word choice because the third dimension is now hypothetical since all observers must be considered two dimensional beings. Such observers cannot directly view the third dimension. We will later see that they can, in principle, measure the curvature of the two dimensional surface, however.

Assume for now that hyperspace is a perfect sphere and that our normal space is the surface of this hypersphere. On the surface of this sphere, we now try to build a two dimensional 'Escher lattice'. Using curved squares will not work in this limited form of hyperspace, but we can fit eight curved equilateral triangles perfectly onto the surface of the sphere, in such a way that they cover the complete surface with no overlaps or gaps. Figure 12.4 shows the four triangles on half of the sphere.

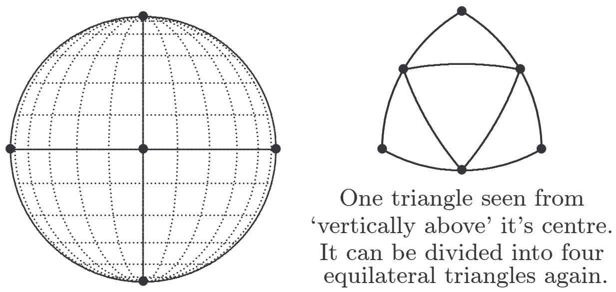

Figure 12.4: The solid lines (left) represent four equilateral triangles fitted onto half of a sphere. All inside angles of each triangle span $\pi/2$ radians, so that inside angles of each triangle add up to $3\pi/2$ radians, instead of the π radians of a flat triangle. Each triangle can be subdivided as shown on the right.

Each of these triangles can be subdivided into four equilateral triangles again, as shown on the right of the figure. This subdivision can be repeated as many times as one wishes.

Every resulting 'connecting rod' will curve towards the centre of the sphere with the same curvature as the original (quarter circumference) rods. In the limit, when the number of subdivisions approach infinity, the infinitesimal little triangles will be indistinguishable from normal flat, Euclidean triangles of the same size.

Their inside angles will add up to π, as near as can be. However, their sides will still have the same curvature, with a radius of curvature equal to the radius of the sphere. The curvature may just not be detectable, due to the tiny size of the triangles in comparison to the radius of the sphere.

The triangles do not have to be tiny—one can just make the radius of the sphere extremely large and the same effect will appear with large triangles.

The reason for laboring this rather trivial point, is that the cosmos may perhaps be somewhat like this, with our observable neighborhood equivalent to one of the triangles, apparently perfectly flat, due to a very large radius of curvature.

If the curvature is positive, the universe is called 'closed', since it is finite, yet unbounded, like the surface of the sphere. In an expanding universe, positive curvature (traditionally) means a universe with an expansion rate that is insufficient to sustain the expansion against the mutual pull of gravity. It is expected to reverse the expansion at some time in the future and collapse towards an infinitely dense state again.

Curvature can also be negative, which can be thought of as the 'inside' of a hyperbola, as will become clearer later. Such a universe is called 'open' and is taken to be unbounded, because it is infinite in size.

Traditionally, negative curvature means an expansion rate that is large enough to sustain the expansion, against the mutual pull of gravity, forever. A universe with zero curvature is traditionally called 'flat' and is also unbounded and infinite.

Zero curvature means that the expansion rate is just large enough so that the expansion rate will decrease asymptotically to zero, so that it will just not collapse. We will later see that these traditional definitions have become a bit blurred lately, due to the discovery that the universe may be 'flat', yet it may actually be expanding at an increasing rate!

The symbology that cosmologists use to describe the curved, expanding universe is illustrated in figure 12.5, where just one of the curved 'Escher rods' is shown—the circular arc between an observer and a distant object.

The time varying *radius of curvature*, denoted by $R(t)$ is equivalent to the radius of the expanding hypersphere. The present radius of curvature is usually written as R, as shown in the figure. The expansion parameter a is now $a(t) = R(t)/R$, meaning the radius of curvature at time t as a fraction of the present radius of curvature.

The comoving distance angle χ has the same meaning as the linear comoving distance r used in the linear 'model', because for any given object, the angle

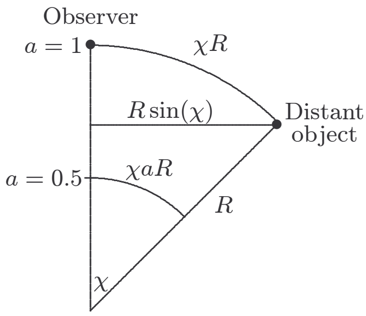

Figure 12.5: The symbology of curved space for a closed cosmos with positive curvature. R is the present radius of curvature, χ the comoving distance and a the expansion factor. For a flat cosmos, R tends to infinity and χ tends to zero, but χR remains finite. $R\sin(\chi)$ is shown because it is used in cosmological equations for the closed cosmos. For the open, negative curvature cosmos, $\sin(\chi)$ is replaced by $\sinh(\chi)$, where $\chi \to i\chi$ and $R \to -iR$.

χ remains constant as $R(t)$ grows with the expansion of the universe. The comoving distance χR is equivalent to the proper length of a stretched Escher rod.

The reader may have found the last paragraph a little confusing and the author can sympathize with that—cosmology can be very confusing, especially since the specialists seem unable to agree on the terminology and symbology for presenting the subject. As an example, two modern textbooks that was consulted, [Peebles] and [Peacock], use different terminology and symbology, as listed in table 12.1.

The differences in symbology are striking and there are some difference in terms as well. It must be said that Prof. Peacock uses 'scale factor' and 'radius of curvature' somewhat interchangeably, while Prof. Peebles does not use the term 'scale factor' at all. In their defense, one must say that contemporary cosmologists suffer under a huge legacy of well established terminology and symbology—well established, but not very consistent.

This text uses the terms and symbology from Prof. Peebles because they are in some sense more intuitively 'accessible', e.g. 'expansion factor' rings a bell that 'normalized scale factor' does not. One might however argue that, judging by the publication dates, Peacock's textbook may be the 'more modern' one.

12.4 An engineering approach

Engineers are meant to take the science that scientists develop and turn it into items of practical use. In cosmology, the engineer's role is usually limited to designing equipment and machinery that can be used to measure the universe at large, so that cosmologists can do their science.

Comparison between some Peacock and Peebles parameters				
	Parameter name	**Peacock**	**Peebles**	**Units**
1	Comoving distance (angle)	r	χ	rad.
2	Present scale factor	R_0	-	Mpc
3	Present radius of curvature	-	R	Mpc
4	Present comoving distance	$R_0 r$	$R\,\chi$	Mpc
5	Present normalized scale factor	a	-	-
6	Present expansion factor	-	a	-
7	Time varying normalized scale factor	$a(t)$	-	-
8	Time varying expansion factor	-	$a(t)$	-
9	Time varying scale factor	R or $a(t)R_0$	-	Mpc
10	Time varying radius of curvature	-	$R(t)$ or $a(t)R$	Mpc

Table 12.1: A comparison between some of the parameter names and symbols used by Peacock and Peebles respectively. Inside the double rows: 2/3, 5/6, 7/8 and 9/10, the two names describe the same parameter. This book uses the Peebles parameter names and symbols.

When working in this type of environment, engineers need to understand what cosmologists are speaking about and probably need to understand the physics and mathematics that cosmologists are using, at least to some degree. Engineers in other environments have little reason to understand the cosmologists; that is unless they have an interest in the cosmological 'machine'.

For their benefit the physics can be presented in a simplified form. The spherical model of fig. 12.5 can be replaced by a flat* model without loosing much of the principles involved.

*Here 'flat' means using non-spherical mathematics and not 'flat' in terms of large scale curvature.

Later chapters will show that this model, using rather simple mathematics, yields very closely the same results than the more complex standard cosmological mathematics.

In the next chapter, we will turn to those parameters, but before we go there, a few introductory words about the units of measurement used in this text.

12.5 Cosmological units

Th SI convention of units is useful for 'ordinary' mass, time and distance, but becomes a bit cumbersome when working with cosmological mass, distance and time. Astronomers like to use the unit 'solar mass' for expressing mass in the cosmos and parsec (pc) for distance. A solar mass is self explanatory and a parsec is the distance at which an object would have an annual parallax of one arc second.

Annual parallax means the peak (not peak to peak) shift in angular position of an object against the very distant stars, observed over a period of one year. So the baseline is the radius of Earth's orbit ($\approx$ 150 million kilometres) and one parsec $\approx \frac{150 \text{ million km}}{\tan(\text{one arc second})} \approx 3.09 \times 10^{13}$ kilometres or 3.2616 light years.

Because the parsec and it's common multiples, e.g. the kiloparsec (kpc) and the Megaparsec (Mpc) are not intuitively understandable by novices, popular books normally express distances in lightyears or orders of magnitude thereof, e.g., million lightyears or billion lightyears, where billion usually means a thousand million.

Since there are traditionally two meanings of 'billion' (USA 10^9 and UK 10^{12}),* this text will adopt the 'more engineering-like' convention of *Giga-*

*Oxford dictionary, 1990: "**billion ....** **2** (now less often, esp. *Brit.*) a million million ...".

lightyear (abbr. Gly $= 10^9$ lightyears) for cosmological distance and *Gigayear* (abbr. Gy $= 10^9$ years) for cosmological time. Cosmology books sometimes use Ga (*Giga-annum)* for the latter.