Cosmic distances

The most useful cosmic distance measure is the comoving distance, which is the present proper distance to an object with observed redshift z_now. For the general case, the comoving distance is obtained by this integration:

$$D_{now} = \frac{978}{H_0} \int_0^{z_{now}} \frac{dz}{\sqrt{\Omega_k (1+z)^2 + \Omega_m (1+z)^3 + \Omega_r (1+z)^4 + \Omega_v)}}$$

in units giga-light-years (Gly). Here H_0 is the Hubble constant in units km/s/Mpc (978 is the conversion factor for H_0 to Gly^{-1}), Ω_k the present curvature density parameter (which is very close to zero, meaning flat space), Ω_m the mass density parameter (including dark matter), Ω_r the radiation energy density parameter and Ω_v the vacuum energy density (dark energy) parameter. Vacuum energy density is also denoted as Ω_Λ, where Λ means Einstein's cosmological constant.

If the present expansion ratio z_now is applied after the integration, the original distance of the object at the time of emission is obtained:

$$D_{then} = \frac{D_{now}}{1 + z_{now}}$$

This is fairly obvious, because $z + 1$ is inversely proportional to the expansion parameter, which tells us by what factor the universe has expanded in the time that the photons took to reach us. D_{then} is also approximately equal to the angular diameter distance of the source.

If the time variable expansion ratio $(1+z)$ is applied inside the integration, the distance that light had to travel to reach us from the source is obtained:

$$D_{light} = \frac{978}{H_0} \int_0^{z_{now}} \frac{dz/(1+z)}{\sqrt{\Omega_k (1+z)^2 + \Omega_m (1+z)^3 + \Omega_r (1+z)^4 + \Omega_v)}}$$

This applies the varying expansion factor all along the path of a photon and adds up the fractional distances to obtain the real path length of the light. D_{light} is also called the lookback distance of an object with redshift z_{now}.

The curves were plotted with the following contemporary values: $H_0 =$ 71 km/s/Mpc, $\Omega_k = 0$, $\Omega_m = 0.27$, $\Omega_r = 8.35 \times 10^{-5}$, $\Omega_v = 0.73$.